We then have the following approximate relation for the speed of sound in the DP on the basis of (4):

$$c = c_3 + (c_2 - c_3)\left\{1 - \left[1 - \left(\frac{r - r_3}{R - r_3}\right)^{(20+\nu)/20}\right]^{2/(\nu+2)}\right\} \text{ for } r_3 \leqslant r \leqslant R. \quad (5)$$

The pressure and density of the DP are determined from the speed of sound via the isentrope. The error in describing the numerical calculations is about 1% for the most sensitive parameter: the pressure.

Therefore, analytic relationships (2)-(5) describe the numerical solution with high accuracy for the distribution of the parameters behind a stationary detonation-wave front in a perfect gas and satisfy the asymptotes of the exact solution.

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DESCRIPTION OF SHOCK-WAVE PROCESSES IN A TWO-PHASE MEDIUM CONTAINING AN INCOMPRESSIBLE PHASE

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The motion of a two-phase medium is analogous to that of a perfect gas with a certain effective adiabatic parameter within the framework of the one-velocity model when the volume proportion of condensed phase is small [1-3]. If on the other hand no constraint is placed on the volume proportion, the basic hydrodynamic equations contain it as a variable additional to those in the analogous gasdynamic equations. This substantially complicates solving the nonstationary hydrodynamic equations and has led to the need to develop the methods given in [4, 5].

Here we propose a method of transforming the variables that leads to complete analogy between the equations for a perfect gas and those for a two-phase medium with any volume occupied by the condensed phase. It is shown that the motion of a two-phase medium in the transformed coordinate system is completely analogous to that of a perfect gas, which means that the methods developed for perfect gases can be used to solve shock-wave problems.

The scope for the method is demonstrated by reference to the strong explosion state in a two-phase medium.

1. Basic Concepts. Consider a homogeneous two-phase medium consisting of condensed and gas phases uniformly distributed in the volume. We assume as follows: 1) The condensed phase is incompressible, 2) the gas obeys the equation of state for a perfect gas with constant values for the specific heats, 3) the partial pressure of the condensed phase is negligibly small, 4) the speeds of the condensed phase and gas are equal, and 5) there is no reaction between the components.

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We introduce the following symbols: p, ρ, u, Γ_0, a_0, E the pressure, density, speed, ratio of the specific heats, equilibrium speed of sound, and internal energy of the two-phase medium, γ and ρ_g the ratio of the specific heats and the density for the gas, ε and d the volume proportion and density of the condensed phase, r the spatial variable, t time, ν a parameter taking the values 1, 2, and 3 correspondingly for planar, cylindrical, and spherical symmetries, and the subscripts 0 and 1 indicating values of the parameters ahead of and at the shock-wave front correspondingly.

With these assumptions, the conservation laws for mass, momentum, and energy give us the following system of equations for the one-dimensional nonstationary motion of the two-phase medium:

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\rho+\frac{\rho}{r^{\nu-1}}\frac{\partial r^{\nu-1}u}{\partial r}=0; \tag{1.1a}$$

$$\rho\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)u+\frac{\partial p}{\partial r}=0; \tag{1.1b}$$

$$\rho\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)E+\frac{p}{r^{\nu-1}}\frac{\partial r^{\nu-1}u}{\partial r}=0. \tag{1.1c}$$

With these assumptions, the equation of state for the two-phase medium may be put as

$$E=p(1-\varepsilon)/[\rho(\Gamma-1)], \tag{1.1d}$$

where Γ is some effective parameter defining a relationship of the form of (1.1d) for a certain range in the thermodynamic parameters. For definiteness we assume that Γ is constant within the relevant flow range. In the particular case $\Gamma=\Gamma_0$, (1.1d) is the equation of state for a thermodynamically equilibrium two-phase mixture [1].

With the above assumptions we have

$$\rho/\rho_0=\varepsilon/\varepsilon_0, \tag{1.2}$$

and so equation of state (1.1d) does not amount to that for a perfect gas. On the other hand, all the other equations in (1.1) are analogous to those describing a perfect gas. We now show that by transforming the variables in (1.1) one can define a coordinate system in which all the definitive equations are completely analogous in form to those for a perfect gas and are not dependent explicitly on ε. The following physical arguments provide a basis for eliminating the volume proportion from (1.1).

In fact, if the condensed phase does not vary in volume in the compressible medium (condition 1) and does not make a contribution to the pressure (condition 3) and moves along the paths of the particles of the compressible phase (condition 4), then one can assume that eliminating the volume occupied by the condensed phase should substantially simplify the mathematical description of the motion.

We transform the variables in (1.1) by denoting them by primes and use the dimensionless coefficients a_i to relate the variables.

Elimination of the volume ε of the incompressible phase means that the mass of the medium should be distributed over the residual volume of the compressible phase, which enables us to relate the densities in the form

$$\rho'=a_1\rho/(1-\varepsilon). \tag{1.3a}$$

The forms of the terms in (1.1) reflecting the effects of spatial symmetry give us for the flow speed that

$$u'=a_2u(r/r')^{\nu-1}. \tag{1.3b}$$

From (1.3a) and (1.3b) we have the relation between the spatial coordinates as

$$dr'=a_3(1-\varepsilon)dr+a_4\varepsilon u'dt. \tag{1.3c}$$

A study of (1.1) indicates the need to transform the time correspondingly:

$$dt'=a_5dt,\quad (\partial t'/\partial r)_t=0. \tag{1.3d}$$

For generality we put

$$p'=a_6p \tag{1.3e}$$

and substitute (1.3) into (1.1) to get as follows with the following relations between the coefficients:

$$a_5 = a_4,\ a_2 = a_6 = 1,\ a_3/a_5 = 1/b = b/a_1,\ \partial b/\partial t + u\partial b/\partial r = 0,\quad b = (r'/r)^{\nu-1} \tag{1.4}$$

as a system of equations analogous in style to those describing the motion of a perfect gas:

$$\left(\frac{\partial}{\partial t'} + u'\frac{\partial}{\partial r'}\right)\rho' + \frac{\rho'}{(r')^{\nu-1}}\frac{\partial (r')^{\nu-1}u'}{\partial r'} = 0,$$
$$\rho'\left(\frac{\partial}{\partial t'} + u'\frac{\partial}{\partial r'}\right)u' + \frac{\partial p'}{\partial r'} = 0, \tag{1.5}$$
$$\rho'\left(\frac{\partial}{\partial t'} + u'\frac{\partial}{\partial r'}\right)\frac{p'}{\rho'(\Gamma-1)} + \frac{p'}{(r')^{\nu-1}}\frac{\partial (r')^{\nu-1}u'}{\partial r'} = 0.$$

It follows from (1.4) that the following are the relations between the parameters in the different systems:

$$\rho' = (r'/r)^{2(\nu-1)}(1-\varepsilon)^{-1}\rho; \tag{1.6a}$$
$$u' = u(r'/r)^{1-\nu}; \tag{1.6b}$$
$$dr' = (r'/r)(1-\varepsilon)dr + (r'/r)^{\nu}\varepsilon u'dt; \tag{1.6c}$$
$$dt' = (r'/r)^{\nu}dt,\quad (\partial t'/\partial r)_t = 0; \tag{1.6d}$$
$$p' = p; \tag{1.6e}$$
$$(\partial/\partial t + u\partial/\partial r)(r'/r)^{\nu-1} = 0; \tag{1.6f}$$
$$E' = p'/[\rho'(\Gamma-1)] = E(r'/r)^{2(1-\nu)}. \tag{1.6g}$$

If there is a linear relationship between the change in volume occupied by the medium and the spatial coordinate (i.e., for planar symmetry, $\nu = 1$), we can put $a_i = 1$. In the general case of arbitrary ν, the coefficients a_i are functions of the symmetry and reflect the nonlinearity in the spatial-coordinate transformation when the volume changes.

Then we can use (1.6) to transform (1.1) into (1.5), whose form is completely analogous with that of the corresponding equations for a perfect gas, which means that within the framework of the one-velocity model for a dispersed medium there exists a coordinate system in which the motion of the two-phase medium is completely analogous to that of a perfect gas. The analogy persists not only for the case where the volume proportion of the condensed phase is small [2] but also in the general case of an arbitrary volume content. Consequently, in order to solve (1.1) and (1.2) it is sufficient to use methods developed for the dynamics of perfect gases to solve (1.5) and use the transformation of (1.6) to solve the initial system.

2. Self-Modeling Flows Containing Shock Waves. The above method has particular advantages in solving stationary problems and in describing a self-modeling flow. We consider the method in relation to the strong explosion stage in a two-phase medium.

Let a finite amount of energy E_0 be instantaneously deposited in an infinitely small volume of a two-phase medium. We restrict ourselves to distances from the explosion source where this wave can be considered as strong, i.e., when one can neglect the initial internal energy of the medium by comparison with E_0, and we consider the propagation of the shock wave moving with speed

$$D = dr_f/dt, \tag{2.1}$$

where $r_f(t)$ is the shock-front coordinate. The solution may be defined in parametric form by the use of dimensionless variables:

$$R = \rho/\rho_0,\ V = u/D,\ P = p/(\rho_0 D^2),\ \eta = r/r_f,\ \chi = r_f/D. \tag{2.2}$$

We supplement (1.1) with the boundary conditions $u = 0$ at $r = 0$ and the following at the shock front [5] $r = r_f$ in order to solve for the point explosion:

$$V = P = 2(1-\varepsilon_0)/(\Gamma+1),\quad R = (\Gamma+1)/(\Gamma-1+2\varepsilon_0) \tag{2.3}$$

together with the integral relation

$$E_0 = \sigma(\nu)\int_0^{r_f}\left(\frac{p(1-\varepsilon)}{\Gamma-1}+\rho\frac{u^2}{2}\right)r^{\nu-1}dr = \sigma(\nu)\rho_0 D^2 r_f^{\nu}\int_0^1\left(\frac{P(1-\varepsilon)}{\Gamma-1}+R\frac{V^2}{2}\right)\eta^{\nu-1}d\eta, \tag{2.4}$$

where $\sigma(\nu) = 2(\nu-1)\pi + (\nu-2)(\nu-3)$.

With these assumptions, the flow described by (1.1) and (1.2) is self-modeling. The method of dimensions [6] shows that R, P, and V are functions of η alone. Then the problem is solved by solving the system

$$(V-\eta)\frac{\partial R}{\partial\eta}+\frac{R}{\eta^{\nu-1}}\frac{\partial\eta^{\nu-1}V}{\partial\eta}=0,\quad (V-\eta)\frac{\partial V}{\partial\eta}-\frac{\nu}{2}V+\frac{1}{R}\frac{\partial P}{\partial\eta}=0, \tag{2.5}$$
$$(1-\varepsilon_0 R)(V-\eta)\frac{\partial P}{\partial\eta}-\nu P(1-\varepsilon_0 R)+\frac{\Gamma P}{\eta^{\nu-1}}\frac{\partial\eta^{\nu-1}V}{\partial\eta}=0.$$

We now derive the dimensionless system corresponding to the primed coordinates and equations relating it to (2.5). We show that the transformation can be written in algebraic form.

Before system (1.5) is rendered dimensionless, we need to incorporate features arising in connection with the time transformation in the primed coordinate system. In the general case, the time varies in different ways at different points in space in the coordinate system (r', t') [denoted also as r'(t')]. It is convenient to transfer to the coordinate system (r', t_1') [denoted as r'(t_1') also] in analyzing the self-modeling flow, where time varies identically for all points, i.e., $t_1' = t$.

We assume that a time dt has passed in the unprimed coordinate system, during which the front has traveled at distance $dr_f = Ddt$, and this time interval at the shock-wave front will correspond to the time $dt_f' = (r_f'/r_f)^{\nu}dt$ the primed coordinate system, during which the front moves a distance $dr_f'(t') = D'dt_f'$. On the other hand, D' can be put as

$$D' = dr_f'(t)/dt. \tag{2.6}$$

The shock-front coordinates are related via the unperturbed-medium parameters

$$dr_f'(t_f') = (r_f'/r_f)(1-\varepsilon_0)\,dr_f. \tag{2.7}$$

Consequently, the relationship between the shock-front speeds is

$$D' = (r_f/r_f')^{\nu-1}(1-\varepsilon_0)D. \tag{2.8}$$

From (2.8) with (2.6) it follows that

$$(r_f'(t)/r_f t)^{\nu} = 1-\varepsilon_0. \tag{2.9}$$

This equation has an obvious interpretation: The ratio of the volumes in which the perturbation propagates in a given time in the different coordinate systems is equal to the ratio of the volume of the gas phase (with ε eliminated from consideration) to the entire volume of the two-phase mixture.

Time varies identically at all points in space in the primed coordinate system, and here the coordinate varies as follows:

$$dr'(t_1') = (r/r')^{\nu}dr'(t') = (r/r')^{\nu-1}(1-\varepsilon)\,dr + \varepsilon u'dt, \tag{2.10}$$

since u' = dr'(t')/dt' = dr'(t)/dt. Equation (2.10) can be used with (1.6a), (1.6b), and (1.6e), which do not vary on passing from (r', t') to (r', t_1'), as a transformation relating to the coordinate systems (r, t) and (r', t_1').

The following equations define the dimensionless flow variables for system (1.5):

$$R' = \rho'/\rho_0',\ V' = u'/D',\ P' = p/(\rho_0'(D')^2),\ \eta' = \frac{r'(t)}{r_f'(t)},\ \chi' = \left(\frac{r_f}{r_f'}\right)^{\nu}\frac{r_f'}{D'},$$

where

$$dt_1' = (r_f/r_f')^{\nu}dt_f' = dt,\ t_1' = t,\qquad \rho_0 = \rho_0'(1-\varepsilon_0)\left(\frac{r_f}{r_f'}\right)^{2(\nu-1)}.$$

We substitute (2.7) into (1.5) and incorporate the conversion from t' to $t_1' = t$ to get a dimensionless system:

$$\chi' \frac{\partial R'}{\partial t_1'} + (V' - \eta') \frac{\partial R'}{\partial \eta'} + \frac{R'}{(\eta')^{\nu-1}} \frac{\partial (\eta')^{\nu-1} V'}{\partial \eta'} = 0,$$

$$\chi' \frac{\partial V'}{\partial t_1'} + (V' - \eta') \frac{\partial V'}{\partial \eta'} + z' V' + \frac{1}{R'} \frac{\partial P'}{\partial \eta'} = 0,$$

$$\chi' \left(\frac{\partial}{\partial t_1'} + (V' - \eta') \frac{\partial}{\partial \eta'} \right) \frac{P'}{\Gamma - 1} + \frac{2z'P'}{\Gamma - 1} + \frac{\Gamma P'}{\Gamma - 1} \frac{1}{(\eta')^{\nu-1}} \frac{\partial (\eta')^{\nu-1} V'}{\partial \eta'} = 0, \tag{2.11}$$

$$z' = \left(\frac{r_f}{r_f'} \right)^{\nu} \frac{r_f'}{D'} \frac{dD'}{dt_1'}.$$

We see that (2.11) differs from (2.5) in having a form analogous to the corresponding system for a perfect gas.

We now consider the transformation relating (2.5) and (2.11). We render (1.6a), (1.6b), and (1.6e) dimensionless without difficulty. We devote particular attention to (2.10). We write the expression for the differentials as

$$dr = \eta dr_f + r_f d\eta, \quad dr' = \eta' dr_f' + r_f' d\eta'. \tag{2.12}$$

When these equations are substituted into (2.10), the terms containing dη and dη' for the self-modeling flow cancel, as will be evident below, i.e., we have

$$d\eta' = \left(\frac{\eta}{\eta'} \right)^{\nu-1} \frac{1 - \varepsilon}{1 - \varepsilon_0} d\eta. \tag{2.13}$$

Then (2.10) after differentiation with respect to time t and use of (2.12) and (2.13) takes the algebraic form

$$\eta' = \eta \frac{1 - \varepsilon}{1 - \varepsilon_0} \left(\frac{\eta}{\eta'} \right)^{\nu-1} + \varepsilon V'. \tag{2.14}$$

We represent the latter relation in differentials and use the first equation in (2.5) to get (2.13).

We supplement (2.14) with (1.6a), and (1.6e) put in dimensionless form:

$$\eta \left(\frac{\eta}{\eta'} \right)^{\nu-1} = \eta' \left(1 + \varepsilon_0 \left(R' \left(\frac{\eta}{\eta'} \right)^{2(\nu-1)} - 1 \right) \right) - \varepsilon_0 R' V' \left(\frac{\eta}{\eta'} \right)^{2(\nu-1)},$$

$$(1 - \varepsilon_0) V' = \left(\frac{\eta}{\eta'} \right)^{\nu-1} V, \; R' = R \left(\frac{\eta'}{\eta} \right)^{2(\nu-1)} \frac{1 - \varepsilon_0}{1 - \varepsilon_0 R}, \; P' = \frac{P}{1 - \varepsilon_0}, \; t_1' = t, \tag{2.15}$$

to give us a relationship between the dimensionless parameters providing one-to-one correspondence between (2.5) and (2.11). We see that (2.15) is algebraic in form.

The boundary conditions at a strong shock-wave front take the following form in the transformed dimensionless system:

$$V' = P' = 2/(\Gamma + 1), \quad R' = (\Gamma + 1)/(\Gamma - 1). \tag{2.16}$$

It is readily seen that (2.16) and (2.3) are related by (2.15), with η = η' = 1 at the wave front.

As there is a one-to-one correspondence between (2.5) and (2.11) and as the transformation does not contain an explicit dependence on time, then if (2.5) has a self-modeling solution, (2.11) will have the same property. Consequently, P', R', V' are independent of time and it follows from (2.4) that $r_f^{\nu} D^2$ = const, and on the basis of (2.9) we have $z' = -\nu/2$.

Therefore, the problem on the strong explosion stage in the transformed coordinate system is that of solving (2.11) without time derivatives for $z' = -\nu/2$; the form of the equations is analogous to that for a perfect gas at the strong stage of explosion. This means that R', V', P' can be determined by using known self-modeling solutions and tabulated data as given in [6-8], while the true distributions of the density R, velocity V, and pressure P can be derived from (2.15).

It is fairly simple to establish analytically the effects of the volume proportion of condensed phase on the laws of motion of the shock wave and the parameters at the shock front without resorting to deriving the distributions of P, V, and R. For this purpose we transform (2.4) by means of (2.15) as follows:

$$E_0 = \sigma(\nu)\rho_0 r_f^{\nu} D^2 \frac{(1-\varepsilon_0)^2}{\Gamma-1}\int_0^1 \left(P' + \frac{\Gamma-1}{2} R'(V')^2\right)(\eta')^{\nu-1} d\eta'.$$

The method of dimensions gives us an equation for the shock-wave path in the form

$$r_f = \left(\frac{E_0}{\alpha\rho_0}\right)^{1/(\nu+2)} \left(\frac{t}{1-\varepsilon_0}\right)^{2/(\nu+2)}, \tag{2.17}$$

where

$$D = \frac{2}{(\nu+2)(1-\varepsilon_0)} \left(\frac{E_0}{\alpha\rho_0}\right)^{0,5} r_f^{-\nu/2},$$

$$\alpha = \frac{4\sigma(\nu)\psi}{(\nu+2)(\Gamma-1)}, \quad \psi = \int_0^1 \left(P' + \frac{\Gamma-1}{2} R'(V')^2\right)(\eta')^{\nu-1} d\eta'.$$

It has been shown [9, 10] that the integral tends to a finite limit for $\Gamma \to 1$ and that for $\Gamma = 1$ we have $\psi = (2\nu)^{-1}$. If we derive ψ from the available theoretical and tabulated data such as in [8], we find that in the entire range in Γ from 1.1 to 1.4 the value of the integral is close to the limiting value $\psi(\Gamma = 1)$ and differs from it by ±3%.

Then the expression for α can be put as

$$\alpha = \left(\frac{2}{\nu+2}\right)^2 \frac{\sigma(\nu)}{2\nu(\Gamma-1)}. \tag{2.18}$$

We use (2.17), (2.18), (2.2), and (2.3) to get a relation between the shock-front pressure and the distance from the explosion center:

$$p = \frac{2(1-\varepsilon_0)}{\Gamma+1}\rho_0 D^2 = \frac{4\nu}{\sigma(\nu)} \frac{\Gamma-1}{\Gamma+1} \frac{E_0}{1-\varepsilon_0} r_f^{-\nu}. \tag{2.19}$$

These equations indicate that the increase in the shock-wave parameters when the medium contains an incompressible phase is due to the increase in the shock-wave speed by a factor $(1-\varepsilon_0)^{-1}$ by comparison with $\varepsilon_0 \to 0$ for a given ratio of the mass concentrations. The shock-wave speed tends to infinity in the limiting case $\varepsilon_0 \to 1$, which from the physical viewpoint is due to the perturbation speed tending to infinity for an incompressible medium.

It follows from (2.19) that the minimum pressure occurs at a given distance from the explosion center in a medium having the maximal shock compressibility for the gas phase [this is defined by $(\Gamma + 1)/(\Gamma - 1)$] with the minimum value of ε_0.

Therefore, in the general case of arbitrary ε the pressure field and the shock-wave velocity in a two-phase medium are dependent not only on the density, the explosion energy, and α [10] but also on the volume proportion of condensed phase. If we neglect ε (in that case $\rho = \varepsilon d + \rho_g$), we get the relations previously derived for the shock-wave parameters in a relaxing two-phase medium [9], while for $\Gamma = \Gamma_0$ we get those for a thermodynamically equilibrium one [6].

In conclusion we note that (1.1) and (1.2) in principle can describe the motion of a wide class of media such as gas-suspension ones, gas–liquid foams, and bubble media. However, it is necessary to check assumptions 1-5 in solving particular problems.

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POROUS-SPECIMEN ADIABATS AND SOLID-COPPER EXPANSION ISENTROPES

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Experiments on shock compression of porous bodies are important in providing information on the thermodynamic parameters of substances at high pressures and temperatures. Specimens of low initial density enable one to obtain higher energies and temperatures with a given specific volume for the shock-compressed material.

It is usually assumed in interpreting experiments with porous specimens that at pressures above the collapse one, at which the density is close to that of the continuous material, the temperature has time to equalize during compression at the shock-wave front, i.e., the states attained are equilibrium ones. However, this requires experimental test.

In [1], the grain size was varied from 0.5 to 100 μm, but no effect on the shock-wave speed was found. This indicates thermal equilibrium for compressed porous specimens.

Here we propose another way of checking the state equilibrium in shock-compressed porous specimens. The thermodynamic parameters of the compressed material are checked from the state parameters derived on unloading a previously compressed solid material. The monitored parameter was the density, and the values were compared for identical pressures and internal energies realized in two different processes. The density comparison for copper shows that the effects of possible deviations from equilibrium during compression of porous specimens do not exceed 1.5% at pressures above 20 GPa.

1. The state of the material in a single-phase system is completely defined by any two thermodynamic parameters if the process is a thermodynamically equilibrium one. If one chooses for example the pressure p and energy E as these, the one can compare the densities ρ or specific volumes $v = 1/\rho$ on porous shock adiabats and expansion isentropes for the solid material for identical p and E to check the consistency in the data and thus to observe possible deviations from thermal equilibrium.

A schematic p–u diagram is used (Fig. 1) to explain the method. In the initial state, the expansion isentrope 2 for a solid specimen on the shock adiabat 1 is characterized by the parameters p_a, u_a, $E_a = u_a^2/2$ ($E = 0$, $p = 0$, $T = T_0$); as the shock-compressed specimen expands, the internal energy decreases. The transition from the hydrodynamic p–u variables to the thermodynamic p–ρ–E ones on the expansion isentrope 2 is provided by calculating the Riemann integrals that express the conservation laws for this type of self-modeling flow:

$$\rho_s(p) = \left[v_a + \int_{u_s}^{u_a} \frac{du}{(dp/du)_s} \right]^{-1}, \quad E_s(p) = E_a - \int_{u_s}^{u_a} p \frac{du}{(dp/du)_s}, \tag{1.1}$$

where the subscripts a and s relate correspondingly to the state on the shock adiabat and on the expansion isentrope. The $E_s(p)$ relation along the isentrope for the solid material can be converted to the p–u relationship 3, on which $u(p) = \sqrt{2E_s(p)}$. In the same coordinates,

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